

# Electrodynamics with an Infrared Scale and PVLAS experiment

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(Dated: February 5, 2008)

We consider an infrared Lorentz violation in connection with recent results of PVLAS experiment. Our analysis is based in a relation that can be established, under certain conditions, between an axial-like-particle theory and electrodynamics with an infrared scale. In the PVLAS case, the conditions imply two dispersion relations such that the infrared scale  $|\vec{\theta}|$ , the inverse axion-photon coupling constant  $M^{-1}$  and the external magnetic field  $\vec{B}$  can be connected through the formula  $|\vec{\theta}| = |\vec{B}|/M$ . Our analysis, which only requires a non-dynamical (auxiliar) axial-like field leads to  $|\vec{\theta}| \leq 5.4 \times 10^{-7}$  meV and  $M^{-1} \sim 1.2 \times 10^{-3}$  GeV<sup>-1</sup>.

## I. INTRODUCTION

Last year the PVLAS collaboration [1] reported that when a linearly polarized laser light crosses a region where there is a transverse magnetic field, a tiny rotation of the polarization plane and a birefringence is observed. This result prompted much activity in high energy physics because it is an unexpected signal within the standard quantum electrodynamics [2].

More recently the PVLAS team [3] did an update of the previous results and although the rotation of the polarization plane was not reconfirmed, a background ellipticity was measured implying a birefringence bound  $10^4$  more bigger than the standard quantum electrodynamics prediction [4, 5, 6] and, therefore, an explanation out of the conventional physics seems still to be necessary.

Several groups have proposed different explanations for the experiment based in axion-like particles (ALP) [8], millicharged particles [9], chameleon fields [11] or refine-

ments of the previous ones.

In this note we would like to study quantum electrodynamics with an infrared scale and analyze the birefringence results in its framework. The possible existence of an infrared scale in quantum field theory has been discussed in the literature from different points of view [7], although in this paper we shall follow [12, 13, 14, 15, 16].

The paper is organized as follow: in section II we present the electrodynamics with an infrared scale and how birefringence emerges; we also establish in this section the conditions under which the infrared modified electrodynamics becomes equivalent to the ALP approach. In section III we interpret the PVLAS experiment in terms of the modified electrodynamics and we establish bounds for the infrared scale and the axion coupling constant. Finally in section IV, we give our conclusions and outlook.

## II. INFRARED MODIFIED ELECTRODYNAMICS

Following [12, 15] the infrared modified electrodynamics is defined through the modified Poisson brackets, *i.e.* instead of considering

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$$\begin{aligned} [A_i(\vec{x}), A_j(\vec{y})] &= 0, \\ [A_i(\vec{x}), \pi_j(\vec{y})] &= \delta_{ij}\delta(\vec{x}, \vec{y}), \\ [\pi_i(\vec{x}), \pi_j(\vec{y})] &= 0, \end{aligned} \quad (1)$$

one writes the following modified commutation relations

$$\begin{aligned} [A_i(\vec{x}), A_j(\vec{y})] &= \kappa_{ij}\delta(\vec{x}, \vec{y}), \\ [A_i(\vec{x}), \pi_j(\vec{y})] &= \delta_{ij}\delta(\vec{x}, \vec{y}), \\ [\pi_i(\vec{x}), \pi_j(\vec{y})] &= \theta_{ij}\delta(\vec{x}, \vec{y}), \end{aligned} \quad (2)$$

where  $\kappa_{ij}$  and  $\theta_{ij}$  are the most general  $3 \times 3$  constant antisymmetric matrices. From the above relations one can see that the canonical dimensions for  $\kappa$  and  $\theta$  are  $(\text{energy})^{-1}$  and  $(\text{energy})^{+1}$  respectively. Therefore, as both scales are introduced as tiny correction to the canonical electrodynamics algebra, they can be identified with an ultraviolet (UV) and an infrared (IR) scale respectively.

In the IR regime –in which we are interested here– we choose  $\theta_{ij} = \epsilon_{ijk}\theta_k$  and  $\kappa_{ij} = 0$  and then the modified Maxwell equations read

$$\begin{aligned} \nabla \cdot \vec{B} &= 0, \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t}, \\ \nabla \cdot \vec{E} &= -\vec{\theta} \cdot \vec{B}, \\ \nabla \times \vec{B} &= -\vec{E} \times \vec{\theta} + \frac{\partial \vec{E}}{\partial t}. \end{aligned} \quad (3)$$

The first two equations in (3) are the standard ones while the other two (*i.e.* the Gauss and Ampere's laws) are changed. Actually these last two equations break explicitly Lorentz invariance and they lead to the two dispersion relations

$$\omega_{\pm}^2 = \vec{k}^2 + \frac{\vec{\theta}^2}{2} \pm \sqrt{(\vec{k}\cdot\vec{\theta})^2 + \frac{1}{4}(\vec{\theta}^2)^2}. \quad (4)$$

The breaking of Lorentz invariance and the two dispersion relations can be understood by noting that the modified Maxwell equations are formally equivalent to the standard ones but in a medium with  $-\vec{\theta} \times \vec{A}$  and  $\vec{\theta} A_0$  playing the role of polarization and magnetization respectively. In such a situation, it is natural to expect Lorentz invariance violation and birefringence.

It is worth noting that equations (3) can be also derived from the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\theta_{\mu}\epsilon^{\mu\nu\rho\lambda}A_{\nu}\partial_{\rho}A_{\lambda}. \quad (5)$$

by taking  $(\theta_{\mu}) = (0, \vec{\theta})$ . Such a Lagrangian also arises in the context of the noncommutative field theories [12] and, moreover, it is the at the basis of the study of Lorentz and

CPT violation developed by Carroll et al [13], Kostelecky et al [14] and others [16].

Interestingly enough, it is not difficult to find a connection between the electrodynamics with an infrared scale discussed above and the ALP model [8]. Indeed, let us consider the Lagrangian for axions coupled to electromagnetism,

$$\mathcal{L}_{ALP} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 + \frac{1}{4M}\varphi \tilde{F}^{\mu\nu}F_{\mu\nu}, \quad (6)$$

with  $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\beta}F_{\rho\beta}$ , the scalar  $\varphi$  is the axion field and  $M^{-1}$  the axion-photon coupling constant. The corresponding equations of motion are

$$\begin{aligned} (\square - m^2)\varphi &= \frac{1}{4M}\tilde{F}^{\mu\nu}F_{\mu\nu}, \\ \nabla \cdot \vec{E} &= -\frac{1}{M}\nabla\varphi \cdot \vec{B}, \\ \nabla \times \vec{B} &= \frac{\partial \vec{E}}{\partial t} + \frac{1}{M}\left(\vec{E} \times \nabla\varphi - \vec{B}\frac{\partial\varphi}{\partial t}\right), \end{aligned} \quad (7)$$

besides the standard ones, *i.e.*  $\nabla \cdot \vec{B} = 0$  and  $\nabla \times \vec{E} = -\partial \vec{B} / \partial t$ .

One should note that the last two eqs. in (3) coincide with the last ones in (7) if one establishes a correspondence

$$\vec{\theta} \leftrightarrow \frac{1}{M}\nabla\varphi. \quad (8)$$

It should be stressed that  $\theta_i$  in the l.h.s. is a constant parameter introduced through the modification of the canonical commutation relations (1) or, what is equivalent, through the addition of a Chern-Simons term as in (5) while  $\varphi$  in the r.h.s is the dynamical axion field.

Furthermore, one can connect the Lagrangian (5) for electrodynamics with an infrared scale with the axion Lagrangian (6) by means of the identity

$$\tilde{F}^{\mu\nu}F_{\mu\nu} = 2\partial_{\mu}(\epsilon^{\mu\nu\rho\lambda}A_{\nu}\partial_{\rho}A_{\lambda}).$$

Then, the term  $\varphi \tilde{F}^{\mu\nu}F_{\mu\nu}$  in (6) can be integrated by parts and written, using the connection (8), in terms of the space-like vector  $\theta_{\mu}$

$$(\partial_{\mu}\varphi)\epsilon^{\mu\nu\rho\lambda}A_{\nu}\partial_{\rho}A_{\lambda} \rightarrow M\theta_{\mu}\epsilon^{\mu\nu\rho\lambda}A_{\nu}\partial_{\rho}A_{\lambda}.$$

Components of  $\theta_{\mu}$  must be small (tiny actually); correspondingly the  $\nabla\varphi$  components must be small and, therefore, the quadratic  $(\nabla\varphi)^2$  term in (6) can be disregarded while the  $m^2\varphi^2$  corresponds in fact to a constant which can be absorbed through a Lagrangian redefinition or as normalization constant in the path integral approach. With all this, the Lagrangians (6) and (5) formally coincide.

Once the connection between Lagrangians (6) and (5) is established, the effects which in the former arise due to a dynamical field can be seen in the later as produced by the infrared parameter,  $\vec{\theta}$ . This means that although the axion interpretation of the PVLAS results seem to be invalidated by recent experiments [18], our approach suggests that there is no need of an axion participation in the birefringence phenomenon reported in PVLAS if the infrared parameter  $|\vec{\theta}|$  is considered. We discuss this issue in the following section.

### III. INTERPRETING THE PVLAS EXPERIMENT

In the PVLAS experiment a linearly polarized photon beam goes through a region where there is an external transverse magnetic field. A non-vanishing ellipticity is observed, which can be attributed to an unusual interaction between photons and the magnetic field. Assuming this, let us interpret the results in the context of the infrared modified electrodynamics presented in the precedent section. To this end, we shall exploit the connection that we established in the precedent section between this model and the ALP theory.

Let us start considering the equations of motion for ALP in the form

$$\square\varphi - \frac{1}{M}\vec{A}\cdot\vec{B} = 0, \quad (9)$$

$$\square\vec{A} + \frac{1}{M}\dot{\varphi}\vec{B} = 0, \quad (10)$$

where  $\vec{B}$  is the external magnetic field and the Coulomb gauge have been used.

From these equations one can derive the following dispersion relations

$$\omega_{\pm}^2 = \vec{k}^2 + \frac{\vec{B}^2}{2M^2} \pm \sqrt{\frac{|\vec{k}|^2|\vec{B}|^2}{M^2} + \frac{|\vec{B}|^4}{4M^4}}, \quad (11)$$

which coincide with (??) if one makes the identification

$$|\vec{\theta}| \leftrightarrow \frac{|\vec{B}|}{M}. \quad (12)$$

However, it should be emphasized that (12) selects only the magnitude of the vectors  $\theta$  and  $\vec{B}$  but not the angles between them.

In summary, the origin of the  $\theta$  modification in the Poisson brackets (2) or in the Lagrangian (5) should be traced back to the introduction of an external magnetic field like that in PVLAS experiment.

Relation (12) expresses the IR scale as a connection between the external magnetic field and the mass scale and, therefore, its magnitudes cannot be computed directly. However using the above dispersion relations, one

can find explicit expressions for two different refractive indices giving rise to birefringence. Indeed, following [17], we chose  $\vec{k}\cdot\vec{\theta} = 0$  and then (??) becomes

$$\omega_+ = |\vec{k}|, \quad \omega_- = \sqrt{|\vec{k}|^2 + |\vec{\theta}|^2}, \quad (13)$$

so that the refractive indices are given by

$$n_+ = 1, \quad n_- = \frac{|\vec{k}|}{\sqrt{|\vec{k}|^2 + |\vec{\theta}|^2}} \approx 1 - \frac{|\vec{\theta}|^2}{2|\vec{k}|^2}. \quad (14)$$

Consequently, the difference of refractive indices  $\Delta n = |n_+ - n_-|$  results in

$$\Delta n = \frac{|\vec{\theta}|^2}{2|\vec{k}|^2}, \quad (15)$$

Since in the infrared modified quantum electrodynamics  $|\vec{\theta}|$  sets the energy scale at which Lorentz invariance could be violated, these relations show that no violation ( $|\vec{\theta}| = 0$ ) corresponds, in terms of the external magnetic field, to  $\vec{B} = 0$ .

Following the alternative route given by the ALP model one should have

$$\Delta n = \frac{|\vec{B}|^2}{2M^2|\vec{k}|^2}. \quad (16)$$

Relations (15)-(16) are of course independent and can be used for computing  $\vec{\theta}$  and  $M$  separately. Indeed, from the PVLAS data we know that

$$k \sim 1.2 \text{ eV}, \quad |\vec{B}| \sim 448.5 \text{ eV}^2,$$

With this and the experimental bound for  $\Delta n$ ,  $\Delta n \leq 10^{-19}$ , one has

$$|\vec{\theta}| \leq 5.4 \times 10^{-7} \text{ meV}, \quad (17)$$

$$M^{-1} \sim 1.2 \times 10^{-3} \text{ GeV}^{-1}. \quad (18)$$

The value for  $M^{-1}$  is three orders above the value obtained by the ALP model [10] and hence the axion does not play, in our approach, any role in the explanation of the PVLAS experiment. In contrast, the bound for  $|\vec{\theta}|$ , not discussed previously within the PVLAS context, is not excluded by any Lorentz violation bound.

### IV. CONCLUSIONS AND OUTLOOK

Quantum electrodynamics calculations, as those presented in refs. [4, 5, 6], lead to results that are four orders of magnitude below the birefringence values measured for example in the PVLAS experiment. This clearly shows that alternative proposals should be investigated

to explain such experiment and in this sense the possibility of axion-like particle production was an attractive one. However, as mentioned before, recent “light shining through a wall” experiments [18] indicate that ALP should be discarded as an explanation for PVLAS results and hence new routes different from that of ALP should be investigated (New experiments in the ALP interpretation will appear the next months, see *e.g* [19, 20, 21]). In fact, in the explanation proposed in this paper, based on a modified version of electrodynamics where an infrared scale  $|\vec{\theta}|$  is introduced, possible axial-like particles are in fact auxiliary and do not take any dynamical role in explaining the PVLAS experiment.

In our model, birefringence results from the modification of the dispersion relations produced by the infrared

scale  $|\vec{\theta}|$ , which in turn is connected to the external magnetic field in which the observed phenomenon takes place. We then conclude that a Lorentz violation such that the bound (17) holds could be at the root of the PVLAS results.

**Acknowledgements:** We would like to thank to O. Bertolami, H. Falomir, E. Fradkin and G. Cantatore by discussions. This work was partially supported by FONDECYT-Chile and CONICYT grants 1050114, 1060079 and 21050196, PIP6160-CONICET, PIC-CNRS/CONICET, BID 1728OC/AR PICT20204-ANPCYT grants and by CIC and UNLP (11/X381 and 11/X450), Argentina.

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[1] E. Zavattini et al. [PVLAS collaboration], *Phys. Rev. Lett.* **96**, 110406 (2006).

[2] See *e.g* W. Dittrich and H. Gies, *Probing the Quantum Vacuum* (Springer, Berlin, 2000); G. V. Dunne, “Heisenberg-Euler effective Lagrangians: Basics and extensions”; M. Roncadelli, *Behind PVLAS*, [[arXiv:hep-ph/0706.4244](#)] and references therein.

[3] E. Zavattini, et al. [PVLAS collaboration], “New PVLAS results and limits on magnetically induced optical rotation and ellipticity in vacuum”, [[arXiv:hep-ph/0706.3419](#)].

[4] S. L. Adler, *Ann. Phys. (N.Y.)* **87**, 599 (1971).

[5] Z. Bialynicka-Birula and I. Bialynicka-Birula, *Phys. Rev. D* **2**, 2341 (1970).

[6] S. L. Adler *J. Phys. A* **40**, F143 (2007); S. Biswas and K. Melnikov, *Phys. Rev. D* **75**, 053003 (2007).

[7] A. G. Cohen, D. B. Kaplan and A. E. Nelson, *Phys. Rev. Lett.* **82**, 4971 (1999); Y.-P. Jing and L. -Z. Fang, *Phys. Rev. Lett.* **73**, 1882 (1994); J. Carmona and J. L. Cortes, *Phys. Rev. D* **65**, 025006 (2002); M. V. Libanov and V. A. Rubakov, *JHEP* **0508**, 001 (2005).

[8] E. Masso and J. Redondo *JCAP* **0509**, 015 (2005); J. Jaeckel, E. Masso, J. Redondo, A. Ringwald and F. Takagashi, “We need lab experiments to look for axion-like particles”, [hep-ph/0605313](#); *ibid*, “The Need for Purely Laboratory-Based Axion-Like Particle Searches”, [hep-ph/0610203](#) and references therein.

[9] R. N. Mohapatra and S. Nasri, [hep-ph/0610068](#), “Reconciling the CAST and PVLAS results”; H. Gies, J. Jaeckel and A. Ringwald, *Phys. Rev. Lett.* **97**, 140402 (2006).

[10] *e.g* M. Ahlers, H. Gies, J. Jaeckel and A. Ringwald, [hep-th/0612098](#), “On the particle interpretation of the PVLAS data: Neutral versus charged particles”; A. Ringwald, “Particle interpretation of the PVLAS data”, [[hep-ph/0704.3195](#)].

[11] P. Brax, C. van Bruck, A. -C. Davis, “Testing Chameleon Theories with Light Propagating through a Magnetic Field”, [[hep-th/0707.2801](#)] and references therein.

[12] J. Gamboa and J. López-Sarrión, *Phys. Rev. D* **71**, 067702 (2005).

[13] S. Carroll, G. Field and R. Jackiw, *Phys. Rev. D* **41**, 1231 (1990).

[14] D. Colladay and V. A. Kostelecky, *Phys. Rev. D* **58**, 116002 (1998); R. Jackiw and V. A. Kostelecky, *Phys. Rev. Lett.* **82**, 3572 (1999); V. A. Kostelecky and R. Lehnert, *Phys. Rev. D* **63**, 065008 (2001).

[15] J. M. Carmona, J. L. Cortés, J. Gamboa and F. Méndez, *JHEP* **0303**, 058 (2003); *ibid* *Phys. Lett. B* **565**, 222 (2003).

[16] A. A. Andrianov, R. Soldati and P. Giacconi, *JHEP* **0202**, 030 (2002).

[17] L. Maiani, R. Petronzio and E. Zavattini, *Phys. Lett. B* **175**, 359 (1986).

[18] C. Robilliard, R. Battesti, M. Fouché, J. Mauchain, A.M. Sautivet and Carlo Rizzo, “No light shining through a wall”, [[hep-ex/0707.1296](#)].

[19] S. J. Chen, H. H. Mei and W. T. Ni, (Q& A Collaboration), [hep-ex/0611050](#).

[20] C. Rizzo (BMV collaboration), 2nd ILIAS-CERN-CAST axion Academic Training 2006, <http://cast.mppmu.mpg.de>.

[21] P. Pugnat *et al.* (OSQAR Collaboration), CERN-SPSC-2006-035, CERN-SPSC-P-331.